Name:

Spring 2017 Math 245 Exam 2

Please read the following directions:
Please write legibly, with plenty of white space. Please print your name on the designated line, similarly to your quizzes (last name(s) in ALL CAPS). Please fit your answers in the designated areas. To get credit, you must also show adequate work to justify your answers. If unsure, show the work. All problems are worth 5-10 points. The use of notes, calculators, or other materials on this exam is strictly prohibited. This exam will begin at 12:40 and will last at most 60 minutes; pace yourself accordingly. Please leave **only** at one of the designated times: 1:00pm, 1:20pm, or 1:40pm. At all other times please stay in your seat (emergencies excepted), to ensure a quiet test environment for others. Good luck!

Problem	Min Score	Your Score	Max Score
1.	5		10
2.	5		10
3.	5		10
4.	5		10
5.	5		10
6.	5		10
7.	5		10
8.	5		10
9.	5		10
10.	5		10
Exam Total:	50		100
Quiz Ave:	50		100
Overall:	50		100

Problem 1. Carefully define the following terms:

a. free variable

b. predicate

c. counterexample

d. Left-to-Right Principle

Problem 2. Carefully define the following terms:

a. Uniqueness Proof Theorem

b. Proof by Contradiction Theorem

c. Proof by Induction Theorem

d. well-ordered set

Problem 3. Simplify $\neg(\exists x, \forall y, \forall z, (x < y) \rightarrow (x < z))$ as much as possible (i.e. where nothing is negated). Do not prove or disprove this statement.

Problem 4. Recall that $\mathbb{R} \setminus \mathbb{Q}$ is the set of irrational numbers. Let $a \in \mathbb{R} \setminus \mathbb{Q}$, $b \in \mathbb{Q}$. Use proof by contradiction to prove that $a + b \in \mathbb{R} \setminus \mathbb{Q}$.

Problem 5. Prove or disprove: $\forall x \in \mathbb{R}, \forall y \in \mathbb{R}, (x < y) \rightarrow \lceil x \rceil \leq \lfloor y \rfloor$.

Problem 6. Let $n \in \mathbb{Z}$. Use the Division Algorithm to prove that $\frac{(n-1)(n+2)}{2} \in \mathbb{Z}$.

Problem 7. Recall the Fibonacci numbers given by $F_0 = 0, F_1 = 1, F_n = F_{n-1} + F_{n-2}$ (for $n \geq 2$). Prove that for all $n \in \mathbb{N}_0$, $F_{n+2} = 1 + \sum_{i=0}^{n} F_i$.

Problem 8. Let $x \in \mathbb{R}$. Prove that $\lfloor x \rfloor$ exists. That is, prove $\exists n \in \mathbb{Z}$, $n \leq x < n+1$.

Problem 9. Use induction to prove $\forall n \in \mathbb{N}$, $\frac{(2n)!}{n!n!} \geq 2^n$.

Problem 10. Let $\mathbb{R}^+$ denote the positive real numbers. Prove that $\forall a \in \mathbb{R}^+$, $\exists b \in \mathbb{R}^+$, $\forall x \in \mathbb{R}$, $|x-2| < b \to |3x-6| < a$.